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501 Ewing Hall
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Professor Colton

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This research project was concerned with the problem of determining the physical and geometric properties of an obstacle by probing with time harmonic acoustic or electromagnetic waves. A new method has been developed to solve problems of this type based on the theory of Herglotz wave functions and complete sets of solutions to the Helmholtz and Maxwell equations. Numerical examples have been given for the case of acoustic waves.

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22a. NAME OF RESPONSIBLE INDIVIDUAL
Dr. Arie Nachman

22b. TELEPHONE (Include Area Code)
(202) 767-4939

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Summary: This research project was concerned with the problem of determining the physical and geometric properties of an obstacle by probing with time harmonic acoustic or electromagnetic waves. A new method has been developed to solve problems of this type based on the theory of Herglotz wave functions and complete sets of solutions to the Helmholtz and Maxwell equations. Numerical examples have been given for the case of acoustic waves.

Professional Personnel

Dr. David Colton — Professor of Mathematics
University of Delaware
Principal Investigator

Dr. Peter Monk — Associate Professor of Mathematics
University of Delaware
Co-Principal Investigator

R.L. Ochs — Graduate Research Assistant
Ph.D. awarded June, 1986
University of Delaware
Thesis Title: The Limited Aperature Problem of Inverse Acoustic Scattering

J. Blöhbaum Graduate Research Assistant

Ph.D awarded June 1988

Universität Göttingen

Thesis Title: Optimization Methods for an Inverse Problem for the Time Harmonic Maxwell Equations

Report on Research

The research of AFOSR Grant 86-0087 was concerned with the problem of determining the physical properties of a scattering obstacle (e.g. shape, density, etc.) from a knowledge of the incident wave and the far field pattern of the scattered wave. Inverse scattering problems of this type can be divided into two major subproblems: the inverse obstacle problem and the inverse medium problem. In the inverse obstacle problem, one is asked to determine the shape of a scattering obstacle of known physical composition, whereas in the inverse medium problem one needs to determine the constitutive parameters in an inhomogeneous medium. For the inverse obstacle problem for time harmonic acoustic waves, it was shown by Colton and Monk ([3]-[6], [11], [12], [17]) that for a fixed frequency the far field patterns of the scattered waves corresponding to different directions of the incident field are all clustered around a hyperplane in $L^2(\partial\Omega)$ where $\partial\Omega$ is the unit sphere. The normal to this hyperplane is, of course, a function of the scattering obstacle D. Hence, one has two distinct optimization schemes for finding D from the far field data: (1) Find D whose far field pattern best fits the measured data and (2) Find D whose associated normal vector in $L^2(\partial\Omega)$ is orthogonal to the measured far field data. Both of these methods have been numerically implemented and tested for numerical efficiency and flexibility ([12], [18], [20]). It should be noted that there is

only one such problem that can be solved exactly: If the acoustic far field pattern is of the form $F(\hat{x}, \alpha) = F_0(\hat{x} \cdot \alpha)$ where $\hat{x}$ is the direction of observation and α the direction of illumination then the obstacle is a ball ([9]). An analogous result, but one which now involves the polarization of the incident field, is also true for electromagnetic waves ([10]).

Optimization methods (with numerical examples) have also been developed for the inverse medium problem for time harmonic acoustic waves ([7], [8], [13], [14], [21]). In the inverse medium problem, a central role is played by a new class of boundary value problems for the Helmholtz equation called <u>interior transmission problems</u>. These problems have eigenvalues which are intimately connected with the nature of the scattering object. Since these eigenvalues can be found from the far field data, such an investigation of the interior transmission problem could lead to new methods for solving the inverse scattering problem.

For the case of an impenetrable medium, the above results have been extended in two different directions by R. Ochs and J. Blöhbaum, students of David Colton. In particular, R. Ochs has shown how to extend the method of Colton and Monk to the case when the far field data is only known in a limited aperature ([18]) and J. Blöhbaum has shown that for the case of the scattering of electromagnetic waves by a perfectly conducting obstacle the far field patterns are all clustered around a hyperplane. A characterization has been given of the normal to this hyperplane and, analogous to the case of acoustic waves, two distinct optimization schemes have been formulated, each of which yields a solution to the inverse obstacle problem for electromagnetic waves ([2], [5], [15]). The extension of these results to the inverse medium problem for electromagnetic waves was given in [16].

The research described above involved a strong interplay between mathematical theory and numerical analysis. Although not directly related to the inverse scattering problem, the papers [1], [19] and [22]-[24] appeared during the time period of this report and can be viewed as "spin-offs" of the main project.

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